H_∞ controller design for continuous networked control systems based on a switched system approach

Linghui Yang[1], Li Gao[1], Zhou Gu[*1,2], Qiuxiang Xue[1]

1. College of Mechanical & Electronic Engineering, Nanjing Forestry University, Nanjing, 210037
E-mail: 15105162854@163.com

2. School of Automation, Southeast University, Nanjing 210096, P. R. China

Abstract: In this paper, the H_∞ controller design problem is investigated for continuous networked control systems (NCSs). Based on a switched system approach, we establish a novel NCSs model, by which the proposed H_∞ controller changes along with the system modes. By using Lyapunov stability theory and Wirtinger-based integral inequality, sufficient conditions for the mean square stability and stabilization of the continuous networked control systems are achieved. Finally, two simulation examples are exploited to demonstrate the correctness and effectiveness of the presented controller design method.

Key Words: Controller design; Networked control systems; Time-varying delay; A switched system approach; Mean square stable

1 INTRODUCTION

Networked control systems(NCSs) is a kind of real-time feedback control systems using communication networks to close control loops. The applications of NCSs can be found in many fields such as automobiles, aircrafts, HVAC systems automation, robotics, manufacturing and industrial process control[1, 2, 3, 4]. The NCSs based on a switched approach introduce some random variables on the basis of NCSs. In this class of systems, the NCSs is modeled as a switched system and it is significant for us to know the probability of a switched system staying in every subsystem because it has an impact on the whole system performance[5]. Compared with the transition probabilities in Markovian jump systems, the probability of a networked stochastic switched system staying in every subsystem is easier to obtain. Therefore, it is necessary to study the performance of networked control systems by using switched system method.

Over the past few decades, this kind of approach has been put forward in[6] and the problem of multiple packet transmission with this method has been studied. Many researchers described the networked control systems with packet loss by the method of switched systems[6],[7],[8]. In[9], Xie analyzed robust stability of a class of discrete-time uncertain switched systems. Besides, when the considered systems are discrete switched system with time delay, the investigations can be seen in [7],[10]. Meanwhile, the study of stability analysis and control for uncertain switched system has been proposed in [11],[12]. The problem of H_∞ output tracking control for networked control systems with random time delays and system uncertainties is investigated in[13]. Stabilization problems of networked control systems (NCSs) with bounded packet losses and random transmission delays are addressed in[14] and [15], they modeled such NCSs as a class of switched systems. [16] modeled the NCS with time-varying transmission intervals as a discrete-time switched linear uncertain system and then obtained bounds for the allowable range of transmission intervals in terms of both minimal and maximal allowable transmission intervals. However, most researchers focus on study of discrete-time networked control systems and the researches on continuous networked control systems with time-varying delay based on switched systems are seldom found. As a result, it still remain challenging.

In this paper, we mainly study the problem of stability analysis and H_∞ controller design for continuous networked control systems with time-varying delay in a switched system method. So, when the system model changes, the design of H_∞ controllers for each subsystems also switch accordingly. Moreover, Wirtinger-based integral inequality which can reduce the conservatism is cited[17]and a numerical example is applied to demonstrate it. Meanwhile, it is challenging that Wirtinger-based integral inequality is used in H_∞ controller design for continuous networked control systems based on a switched system approach. Then in section 3, we get the linear matrix equalities(LMIs) which make the networked control systems mean square stable with a H_∞ norm bound γ and design the H_∞ controller by applying Lyapunov functional approach and switched systems method. In section 4, we show that it is more effective to use the suggested method than other methods by two numerical examples and simulation figures. At last, in section 5, the conclusion is obtained.

*Corresponding author. Email: guzhou@njfu.edu.cn.

This work is supported by National Nature Science Foundation of China(Grant No. 61473156), Research Fund for the Doctoral Program of Higher Education of China (Grant No. 20133204120018) and the National Science Foundation for Post-doctoral Scientists of China (Grant No. 2014M551487) and Jiangsu Post-doctor Grant(No. 1301009A).

Notations: Throughout this paper, the notation Z^+ represents positive integer set and $\mathbb{R}^n$ represents the n-dimensional Euclidean space. The notation $S > 0 (S < 0)$ is used to denote a positive definite(negative definite) matrix. $\|\cdot\|$ denotes the spectral norms of matrices or the Euclidean norm for vectors. $E[\cdot]$ represents mathematical expectation and $r(t)$ represents switching variable. I denotes the identity matrix. In symmetric block matrices, $*$ is used as ellipsis for terms induced by symmetry and for any square matrices A and B, we define diag{A,B}=$\begin{bmatrix} A & * \\ 0 & B \end{bmatrix}$.

2 PRELIMINARIES

In this section, a switched system approach is proposed, which apply to stability analysis for continuous networked control systems with time-varying delay. Then an assumption and two lemmas are given, which are helpful to H_∞ stability analysis and controller design. In non-ideal conditions, the continuous networked control system will be affected by the external interference factors, so the continuous networked control system with time-varying delay based on a switched system approach can be represented as:

$$\begin{cases} \dot{x}(t) = A_{\sigma(t)}x(t) + B_{\sigma(t)}u(t) + D_{\sigma(t)}\omega(t) \\ z(t) = C_{\sigma(t)}x(t) \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^n$ is the control input, $\omega(t) \in \ell_2[0,\infty)$ is disturbance input vector, $z(t) \in \mathbb{R}^m$ is controlled output vector,In this paper, the sampling period of NCSs is h, $i_k(k = 1,2,3...)$ are some positive integers such that $\{i_1, i_2, i_3, ...\} \subset \{0, 1, 2, ...\}$. τ_k is the i_kth packet's transmission time and $\sigma(t)$ is a switching signal which is right continuous. Then the following networked state feedback controller is designed like this: $u(t^+) = K_{\sigma(t)}x(i_kh), t \in \{i_kh + \tau_k, k = 1, 2, ...\}$, where $K_{\sigma(t)}$ is controller gain to be determined. Define $\eta(t) = t - i_kh, t \in [i_kh + \tau_k, i_{k+1}h + \tau_{k+1})$ which stands for time-varying delay of this system and there are η_m and η_M which make $\eta_m \le \eta(t) \le \eta_M$. Then the controller for system(1) can be represented as $u(t) = K_{\sigma(t)}x(t - \eta(t))$. So, the system (1) can be changed to:

$$\begin{cases} \dot{x}(t) = A_{\sigma(t)}x(t) + B_{\sigma(t)}K_{\sigma(t)}x(t-\eta(t)) \\ \qquad + D_{\sigma(t)}\omega(t), t \in [i_kh + \tau_k, i_{k+1}h + \tau_{k+1}) \\ z(t) = C_{\sigma(t)}x(t) \\ x(t) = \phi(t), t \in [t_0 - \eta_M, t_0 - \eta_m] \end{cases} \tag{2}$$

where the delay function $\eta(t)$ is unknown and time-varying, $\phi(t)$ is the initial condition of state. Because in (2), $\sigma(t)$ is a switching variable, for $\sigma(t) = i (i \in \Omega = 1, 2, ..., N)$, A_i, B_i, C_i, D_i are constant matrix with appropriate dimensions of the ith subsystem and K_i is the state feedback controller gain of the ith subsystem to be designed.
For facilitating theoretical development, we make the following assumption which is common in using a switched system approach.

Assumption 1 *In this paper, the probability of a continuous networked control system staying in each subsystem is assumed to be known, that is to say,* $Pr\{\sigma(t) = i | t \in Z^+, i \in \Omega = 1, 2, ..., N\} = \bar{\alpha}_i$, *where* $\bar{\alpha}_i \in [0, 1]$ *and* $\bar{\alpha}_i$ *represents the probability of a continuous networked control system staying in* i*th subsystem. Assuming that a set of stochastic variable* $\alpha_i(t)$ *are defined as*

$$\alpha_i(t) = \begin{cases} 1, \sigma(t) = i \\ 0, \sigma(t) \neq i \end{cases}$$

So, we can see that the expectation of $\alpha_i(t)$ *is* $\bar{\alpha}_i$. $\alpha_i(t)$ *and* $\bar{\alpha}_i$ *satisfy* $\sum_{i=1}^N \alpha_i(t) = 1, \sum_{i=1}^N \bar{\alpha}_i = 1$.

Remark 1 *The* $\bar{\alpha}_i$ *can be obtained through statistical method like this:* $\bar{\alpha}_i = \lim_{a\to\infty} \frac{k_i}{a}$, *where* k_i *is the times of* $\sigma(a) = i$ *in the interval* $[1, a]$, $a \in Z^+$.

Remark 2 *Similar to [18], it is also assumed that* $\alpha_i(t)$ *in assumption 1 is a Bernoulli-distributed sequence. Notice that the variance of* $\alpha_i(t)$ *is* $D\{\alpha_i(t)\} = \bar{\alpha}_i(1 - \bar{\alpha}_i)$.

Based on the above assumption, the model of the continuous networked control system which contains N subsystems can be written as:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{N} \alpha_i(t)[A_ix(t) + B_iK_ix(t - \eta(t)) \\ \qquad + D_i\omega(t)], t \in [i_kh + \tau_k, i_{k+1}h + \tau_{k+1}) \\ z(t) = \sum_{i=1}^{N} \alpha_i(t)C_ix(t) \\ x(t) = \phi(t), t \in [t_0 - \eta_M, t_0 - \eta_m] \end{cases} \tag{3}$$

For the convenience of proof of stability for continuous networked control systems with time-varying delay, we refer to the following two lemmas.

Lemma 1 *[17] For given positive integers n,m,a scalar* α *in the interval* $(0, 1)$, *a given* $n \times n - matrix R > 0$, *two matrices* W_1 *and* W_2 *in* $\mathbb{R}^{n\times m}$. *Define, for all vector* ξ *in* $\mathbb{R}^m$, *the function* $\Theta(\alpha, R)$ *given by:* $\Theta(\alpha, R) = \frac{1}{\alpha}\xi^TW_1^TRW_1\xi + \frac{1}{1-\alpha}\xi^TW_2^TRW_2\xi$
Then, if there exists a matrix H *in* $\mathbb{R}^{n\times n}$ *such that* $\begin{bmatrix} R & H \\ * & R \end{bmatrix} > 0$,*then the following inequality holds*

$$\min_{\alpha\in(0,1)} \Theta(\alpha, R) \ge \begin{bmatrix} W_1\xi \\ W_2\xi \end{bmatrix}^T \begin{bmatrix} R & H \\ * & R \end{bmatrix} \begin{bmatrix} W_1\xi \\ W_2\xi \end{bmatrix}.$$

This lemma will be useful to obtain stability conditions for continuous networked control systems with time-varying delays.

Lemma 2 *[17] For a given* $n \times n$*-matrix* $R > 0$, *the following inequality holds for all continuously differentiable function* x *in* $[a, b] \to \mathbb{R}^n$:
$\int_a^b \dot{x}^T(u)R\dot{x}(u)du \ge \frac{1}{b-a}(x(b)-x(a))^TR(x(b)-x(a)) + \frac{3}{b-a}\tilde{\Omega}^TR\tilde{\Omega}$,
where $\tilde{\Omega} = x(b) + x(a) - \frac{2}{b-a}\int_a^b x(u)du$.

3 H_∞ STABILITY ANALYSIS and CONTROLLER DESIGN

In this section, we will mainly study H_∞ stability analysis and controller design of system (3). First of all, we will develop theorem1 for the system (4) and then when the continuous networked control system only has one subsystem, the corollary is developed in section 3.1. At last, the H_∞ controller for the continuous networked control system which is based on a switched system approach is designed in section 3.2.
To facilitate the research, in system (3), we can define

$$\mathcal{A}_i = \begin{bmatrix} A_i & B_iK_i & 0 & 0 & 0 & D_i \end{bmatrix},$$

$$\zeta(t) = \begin{bmatrix} x(t) \\ x(t-\eta(t)) \\ x(t-\eta_M) \\ \frac{1}{\eta(t)}\int_{t-\eta(t)}^{t} x(s)ds \\ \frac{1}{\eta_M-\eta(t)}\int_{t-\eta_M}^{t-\eta(t)} x(s)ds \\ \omega(t) \end{bmatrix}^T,$$

Then, combing the above formula which we have defined with system (3), we can get the following system's equations:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{N} \alpha_i(t)\mathcal{A}_i\zeta(t), \\ \quad t \in [i_kh+\tau_k, i_{k+1}h+\tau_{k+1}) \\ z(t) = \sum_{i=1}^{N} \alpha_i(t)C_ix(t) \\ x(t) = \phi(t), t \in [t_0-\eta_M, t_0-\eta_m] \end{cases} \tag{4}$$

Remark 3 *Define the $\mathcal{A}_i$ and $\zeta(t)$ like this is for the convenience of using Lyapunov functional method. In addition, in the following proof of theorem 1, we will divide the integral $[t-\eta_M, t]$ into two parts $[t-\eta_M, t-\eta(t)]$ and $[t-\eta(t), t]$.*

In this section, our main purpose is designing H_∞ controller and make the two conditions establish in this following definition.

Definition 1 *System (4) is robust mean square stable with a H_∞ norm bound γ if the following hold:*
(1) When $\omega(t) = 0$, system (4) is robust mean square stable;
(2) For a scalar $\gamma > 0$ and zero initial condition, $z(k)$ satisfies $E\{\sum_{t=0}^{\infty}\|z(t)\|_2\} \leq \gamma E\{\sum_{t=0}^{\infty}\|\omega(t)\|_2\}$

3.1 Stability Analysis

According to the above analysis, now the stability conditions of the continuous networked control system (4) will be given.

Theorem 1 *For a given positive constant γ and matrices $K_i(i \in \Omega)$, assume that there exist matrices $P > 0$, $Q > 0$, $R > 0$ and H such that the following LMIs is satisfied for $\eta(t) \in \{\eta_m, \eta_M\}$*

$$\Psi_1 = \begin{bmatrix} \Xi_{11} & * & * & * & * & * & * \\ \Xi_{21} & \Xi_{22} & * & * & * & * & * \\ \Xi_{31} & \Xi_{32} & \Xi_{33} & * & * & * & * \\ \Xi_{41} & \Xi_{42} & \Xi_{43} & \Xi_{44} & * & * & * \\ \Xi_{51} & \Xi_{52} & \Xi_{53} & \Xi_{54} & \Xi_{55} & * & * \\ \Xi_{61} & 0 & 0 & 0 & 0 & -\gamma^2 I & * \\ \Xi_{71} & \Xi_{72} & 0 & 0 & 0 & \Xi_{76} & \Xi_{77} \end{bmatrix} < 0 \tag{5}$$

$$\begin{bmatrix} \hat{R} & H \\ * & \hat{R} \end{bmatrix} > 0, \tag{6}$$

where

$$\Xi_{11} = \bar{\alpha}_iA_i^TP + \bar{\alpha}_iPA_i + Q - \frac{4}{\eta_M}R + \bar{\alpha}_iC_i^TC_i,$$
$$\Xi_{21} = \bar{\alpha}_iK_i^TB_i^TP - \frac{1}{\eta_M}(2R + H_1 + H_2 + H_2^T + H_3),$$
$$\Xi_{22} = -\frac{1}{\eta_M}(8R - 2H_1 + 2H_3), \hat{R} = diag\{R, 3R\},$$
$$\Xi_{31} = -\frac{1}{\eta_M}(-H_1 + H_2 - H_2^T + H_3),$$
$$\Xi_{32} = -\frac{1}{\eta_M}(2R + H_1 - H_2 - H_2^T + H_3),$$
$$\Xi_{33} = -Q - \frac{4}{\eta_M}R, \Xi_{41} = \frac{6}{\eta_M}R,$$
$$\Xi_{42} = \frac{1}{\eta_M}(6R + 2H_2 + 2H_3), \Xi_{43} = -\frac{1}{\eta_M}(2H_2 - 2H_3),$$
$$\Xi_{44} = -\frac{12}{\eta_M}R, \Xi_{51} = \frac{1}{\eta_M}(2H_2 + 2H_3),$$
$$\Xi_{52} = -\frac{1}{\eta_M}(2H_2 - 2H_3 - 6R), \Xi_{53} = \frac{6}{\eta_M}R,$$
$$\Xi_{54} = -\frac{4}{\eta_M}H_3, \Xi_{55} = -\frac{12}{\eta_M}R, \Xi_{61} = \bar{\alpha}_iD_i^TP,$$
$$\Xi_{71} = \bar{\alpha}_i\eta_MRA_i, \Xi_{72} = \bar{\alpha}_i\eta_MRB_iK_i, \Xi_{76} = \bar{\alpha}_i\eta_MRD_i,$$
$$\Xi_{77} = -\eta_M\bar{\alpha}_iR, H = \begin{bmatrix} H_1 & * \\ H_2 & H_3 \end{bmatrix}.$$

Then the system (4) is mean square stable with an H_∞ norm bound γ for all delay function $\eta(t)$.

Proof: Consider the functional given by

$$V(\eta, x_t, \dot{x}_t) = x^T(t)Px(t) + \int_{t-\eta_M}^{t} x^T(s)Qx(s)ds + \int_{-\eta_M}^{0}\int_{t+s}^{t} \dot{x}^T(v)R\dot{x}(v)dvds,$$

This functional is positive definite since $P > 0$, $Q > 0$, and $R > 0$. The expectation is

$$\begin{aligned} &E\{\mathcal{L}(V) + z^T(t)z(t) - \gamma^2\omega^T(t)\omega(t)\} \\ &= E\{2x^T(t)P\dot{x}(t) + x^T(t)Qx(t) \\ &- x^T(t-\eta_M)Qx(t-\eta_M) + \eta_M\dot{x}^T(t)R\dot{x}(t) \\ &- \int_{t-\eta_M}^{t} \dot{x}^T(s)R\dot{x}(s)ds \\ &+ z^T(t)z(t) - \gamma^2\omega^T(t)\omega(t)\} \end{aligned} \tag{7}$$

Note that

$$E\{\alpha_i(t)\alpha_i(t)\} = E\{\alpha_i^2(t)\} = D\{\alpha_i(t)\} + E\{\alpha_i(t)\}^2 = \bar{\alpha}_i(1-\bar{\alpha}_i) + \bar{\alpha}_i^2 = \bar{\alpha}_i$$

then (7) can be changed to

$$\begin{aligned} &E\{\mathcal{L}(v) + z^T(t)z(t) - \gamma^2\omega^T(t)\omega(t)\} \\ &= E\{\sum_{i=1}^N \bar{\alpha}_i\zeta^T(t)\mathcal{A}_i^T Px(t) + \sum_{i=1}^N \bar{\alpha}_i x^T(t)P\mathcal{A}_i\zeta^T(t) \\ &+ x^T(t)Qx(t) - x^T(t-\eta_M)Qx(t-\eta_M) \\ &- \int_{t-\eta_M}^t \dot{x}^T(s)R\dot{x}(s)ds + \sum_{i=1}^N \bar{\alpha}_i x^T(t)C_i^T C_i x(t) \\ &- \gamma^2\omega^T(t)\omega(t) + \eta_M \sum_{i=1}^N \bar{\alpha}_i\zeta^T(t)\mathcal{A}_i^T R\mathcal{A}_i\zeta(t)\} \end{aligned} \quad (8)$$

First of all, we deal with the item of integral $\int_{t-\eta_M}^t \dot{x}^T(s)R\dot{x}(s)ds$ and dividing the integral into two integrals, which including $[t-\eta_M, t-\eta(t)]$ and $[t-\eta(t), t]$. Then, applying Lemma 1 to two integrals. So, it yields

$$\begin{aligned} &-\int_{t-\eta_M}^t \dot{x}^T(s)R\dot{x}(s)ds \\ &\leq -\zeta^T(t)(\frac{1}{\eta(t)}F_1^T\hat{R}F_1 + \frac{1}{\eta_M-\eta(t)}F_2^T\hat{R}F_2)\zeta(t) \end{aligned} \quad (9)$$

where

$$F_1 = \begin{bmatrix} I & -I & 0 & 0 & 0 & 0 \\ I & I & 0 & -2I & 0 & 0 \end{bmatrix},$$

$$F_2 = \begin{bmatrix} 0 & I & -I & 0 & 0 & 0 \\ 0 & I & I & 0 & -2I & 0 \end{bmatrix}.$$

Then, applying Lemma 2,we can see that if there exists a matrix $H \in \mathbb{R}^{n\times n}$ such that $\begin{bmatrix} \hat{R} & H \\ * & \hat{R} \end{bmatrix} > 0$, it will yield

$$\begin{aligned} &\frac{1}{\eta(t)}F_1^T\hat{R}F_1 + \frac{1}{\eta_M-\eta(t)}F_2^T\hat{R}F_2 \\ &= \frac{1}{\eta_M}[\frac{1}{\eta(t)/\eta_M}F_1^T\hat{R}F_1 + \frac{1}{[1-\eta(t)]/\eta_M}F_2^T\hat{R}F_2] \\ &\geq \frac{1}{\eta_M}\begin{bmatrix} F_1 \\ F_2 \end{bmatrix}^T \begin{bmatrix} \hat{R} & H \\ * & \hat{R} \end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \end{bmatrix} \end{aligned} \quad (10)$$

So, combining the inequality (9) and (10), we can obtain $-\int_{t-\eta_M}^t \dot{x}^T(s)R\dot{x}(s)ds \leq -\frac{1}{\eta_M}\zeta^T(t)\begin{bmatrix} F_1 \\ F_2 \end{bmatrix}^T \begin{bmatrix} \hat{R} & H \\ * & \hat{R} \end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \end{bmatrix}\zeta(t)$.

Then, using Schur complement in the right side of equality (8), it yields

$$\begin{bmatrix} \Xi_{11} & * & * & * & * & * & * \\ \Xi_{21} & \Xi_{22} & * & * & * & * & * \\ \Xi_{31} & \Xi_{32} & \Xi_{33} & * & * & * & * \\ \Xi_{41} & \Xi_{42} & \Xi_{43} & \Xi_{44} & * & * & * \\ \Xi_{51} & \Xi_{52} & \Xi_{53} & \Xi_{54} & \Xi_{55} & * & * \\ \Xi_{61} & 0 & 0 & 0 & 0 & -\gamma^2 I & * \\ A_i & B_iK_i & 0 & 0 & 0 & D_i & \check{\Xi}_{77} \end{bmatrix} < 0 \quad (11)$$

where $\check{\Xi}_{77} = -\eta_M^{-1}\bar{\alpha}_i^{-1}R^{-1}$, then pre- and post-multiple both sides of matrix with diag $\{I, I, I, I, I, I, R\}$ and its transpose, then pre- and post-multiple with η_M and $\bar{\alpha}_i$. Then the inequality below can be obtained: $E\{\mathcal{L}(V) + z^T(t)z(t) - \gamma^2\omega^T(t)\omega(t)\} \leq E\{\zeta^T(t)\Psi_1\zeta(t)\}$. Finally, $E\{\zeta^T(t)\Psi_1\zeta(t)\} \leq 0$ if there exists a matrix H such that $\begin{bmatrix} \hat{R} & H \\ * & \hat{R} \end{bmatrix} > 0$ and if $\Psi_1 < 0$.Under zero initial conditions, when $t \to \infty$ and recalling Definition 1, the proof is completed.

Theorem 1 analyze the stability of continuous networked control system (4) by using Lyapunov functional method and a switched system approach. And according to the above analysis, the following corollary is concluded. In inequality (5), letting $\bar{\alpha}_i = 1$, the Theorem 1 is still establish.

Corollary 1 *For a given positive constant γ and a matrix K, assume that there exist matrices $P > 0, Q > 0, R > 0$ and H such that the following LMIs is satisfied for $\eta(t) \in \{\eta_m, \eta_M\}$*

$$\begin{bmatrix} \bar{\Xi}_{11} & * & * & * & * & * & * \\ \bar{\Xi}_{21} & \Xi_{22} & * & * & * & * & * \\ \Xi_{31} & \Xi_{32} & \Xi_{33} & * & * & * & * \\ \Xi_{41} & \Xi_{42} & \Xi_{43} & \Xi_{44} & * & * & * \\ \Xi_{51} & \Xi_{52} & \Xi_{53} & \Xi_{54} & \Xi_{55} & * & * \\ D_i^T P & 0 & 0 & 0 & 0 & -\gamma^2 I & * \\ \bar{\Xi}_{71} & \bar{\Xi}_{72} & 0 & 0 & 0 & \bar{\Xi}_{76} & -\eta_M R \end{bmatrix} < 0, \quad (12)$$

$$\begin{bmatrix} \hat{R} & H \\ * & \hat{R} \end{bmatrix} > 0 \quad (13)$$

where

$$\bar{\Xi}_{11} = A^T P + PA + Q - \frac{4}{\eta_M}R + C^T C,$$

$$\bar{\Xi}_{21} = K^T B^T P - \frac{1}{\eta_M}(2R + H_1 + H_2 + H_2^T + H_3),$$

$$\bar{\Xi}_{71} = \eta_M RA, \bar{\Xi}_{72} = \eta_M RBK, \bar{\Xi}_{76} = \eta_M RD.$$

Then the continuous networked control system is mean square stable with an H_∞ norm bound γ for all delay function $\eta(t)$.

The proof is similar to theorem 1, it is omitted for brevity.

3.2 H_∞ Controller Design

In this section, the H_∞ controller for the continuous networked control system with time-varying delay will be designed.

Theorem 2 *For some given positive constants γ and ε_0, assume that there exist matrices $\tilde{P} > 0$, $\tilde{Q} > 0$, $\tilde{R} > 0$ and $\tilde{H}$ such that the following LMIs is satisfied for $\eta(t) \in \{\eta_m, \eta_M\}$*

$$\Psi_2 = \begin{bmatrix} \Theta_{11} & * \\ \bar{\alpha}_i C_i X & -\bar{\alpha}_i I \end{bmatrix} < 0 \quad (14)$$

$$\begin{bmatrix} \check{R} & \tilde{H} \\ * & \check{R} \end{bmatrix} > 0 \quad (15)$$

where

$$\Theta_{11} = \begin{bmatrix} \tilde{\Xi}_{11} & * & * & * & * & * & * \\ \tilde{\Xi}_{21} & \tilde{\Xi}_{22} & * & * & * & * & * \\ \tilde{\Xi}_{31} & \tilde{\Xi}_{32} & \tilde{\Xi}_{33} & * & * & * & * \\ \tilde{\Xi}_{41} & \tilde{\Xi}_{42} & \tilde{\Xi}_{43} & \tilde{\Xi}_{44} & * & * & * \\ \tilde{\Xi}_{51} & \tilde{\Xi}_{52} & \tilde{\Xi}_{53} & \tilde{\Xi}_{54} & \tilde{\Xi}_{55} & * & * \\ \tilde{\Xi}_{61} & 0 & 0 & 0 & 0 & -\gamma^2 I & * \\ \tilde{\Xi}_{71} & \tilde{\Xi}_{72} & 0 & 0 & 0 & \tilde{\Xi}_{76} & \tilde{\Xi}_{77} \end{bmatrix}$$

$$\tilde{\Xi}_{11} = \bar{\alpha}_i X A_i^T + \bar{\alpha}_i A_i X + \tilde{Q} - \frac{4}{\eta_M}\tilde{R},$$

$$\tilde{\Xi}_{21} = \bar{\alpha}_i Y_i^T B_i^T - \frac{1}{\eta_M}(2\tilde{R} + \tilde{H}_1 + \tilde{H}_2 + \tilde{H}_2^T + \tilde{H}_3),$$

$$\tilde{\Xi}_{22} = -\frac{1}{\eta_M}(8\tilde{R} - 2\tilde{H}_1 + 2\tilde{H}_3), \tilde{H} = \begin{bmatrix} \tilde{H}_1 & * \\ \tilde{H}_2 & \tilde{H}_3 \end{bmatrix}$$

$$\tilde{\Xi}_{31} = -\frac{1}{\eta_M}(-\tilde{H}_1 + \tilde{H}_2 - \tilde{H}_2^T + \tilde{H}_3),$$

$$\tilde{\Xi}_{32} = -\frac{1}{\eta_M}(2\tilde{R} + \tilde{H}_1 - \tilde{H}_2 - \tilde{H}_2^T + \tilde{H}_3),$$

$$\tilde{\Xi}_{33} = -\tilde{Q} - \frac{4}{\eta_M}\tilde{R}, \tilde{\Xi}_{41} = \frac{6}{\eta_M}\tilde{R}, \check{R} = diag\{\tilde{R}, 3\tilde{R}\}$$

$$\tilde{\Xi}_{42} = \frac{1}{\eta_M}(6\tilde{R} + 2\tilde{H}_2 + 2\tilde{H}_3), \tilde{\Xi}_{43} = -\frac{1}{\eta_M}(2\tilde{H}_2 - 2\tilde{H}_3),$$

$$\tilde{\Xi}_{44} = -\frac{12}{\eta_M}\tilde{R}, \tilde{\Xi}_{51} = \frac{1}{\eta_M}(2\tilde{H}_2 + 2\tilde{H}_3),$$

$$\tilde{\Xi}_{52} = -\frac{1}{\eta_M}(2\tilde{H}_2 - 2\tilde{H}_3 - 6\tilde{R}), \tilde{\Xi}_{53} = \frac{6}{\eta_M}\tilde{R},$$

$$\tilde{\Xi}_{54} = -\frac{4}{\eta_M}\tilde{H}_3, \tilde{\Xi}_{55} = -\frac{12}{\eta_M}\tilde{R}, \tilde{\Xi}_{61} = \bar{\alpha}_i D_i^T,$$

$$\tilde{\Xi}_{71} = \bar{\alpha}_i \eta_M A_i X, \tilde{\Xi}_{72} = \bar{\alpha}_i \eta_M B_i Y_i, \tilde{\Xi}_{76} = \bar{\alpha}_i \eta_M D_i,$$

$$\tilde{\Xi}_{77} = \bar{\alpha}_i \eta_M(-2\varepsilon_0 X + \varepsilon_0^2 \tilde{R}).$$

Then the system (3) is robust mean square stable with a H_∞ norm bound γ for all delay function $\eta(t)$ and the controller feedback gains $K_i = Y_i X^{-1}$.

Proof: First of all, pre- and post-multiply both sides of (11) with diag $\{I, I, I, I, I, I, P\}$ and its transpose, then it yields

$$\begin{bmatrix} \Xi_{11} & * & * & * & * & * & * \\ \Xi_{21} & \Xi_{22} & * & * & * & * & * \\ \Xi_{31} & \Xi_{32} & \Xi_{33} & * & * & * & * \\ \Xi_{41} & \Xi_{42} & \Xi_{43} & \Xi_{44} & * & * & * \\ \Xi_{51} & \Xi_{52} & \Xi_{53} & \Xi_{54} & \Xi_{55} & * & * \\ \Xi_{61} & 0 & 0 & 0 & 0 & -\gamma^2 I & * \\ PA_i & PB_iK_i & 0 & 0 & 0 & PD_i & \hat{\Xi}_{77} \end{bmatrix} < 0 \quad (16)$$

where $\hat{\Xi}_{77} = -\eta_M^{-1}\bar{\alpha}_i^{-1}PR^{-1}P$,
then letting $X = P^{-1}$,$XRX = \tilde{R}$,$XH_1X = \tilde{H}_1$,$XH_2X = \tilde{H}_2$,$XH_3X = \tilde{H}_3$,$XQX = \tilde{Q}$ and $Y_i = K_iX$.
Due to

$$(\varepsilon_i R_i - P_i)R_i^{-1}(\varepsilon_i R_i - P_i) \geq 0$$

where ε_i is a given positive constant, then it is true that

$$-P_i R_i^{-1} P_i \leq -2\varepsilon_i P_i + \varepsilon_i^2 R_i \quad (17)$$

Subsequently, combining (16), (17) and Theorem 1 and pre- and post-multiply both sides with diag {X,X,X,X,X,I,X,I,I} and its transpose. Finally, based on the analysis above, using Schur complement, the matrix inequality (14) and (15) can be easily obtained from (5) and (6). This completes the proof.

4 EXAMPLES

In this section, two examples are given to demonstrate the effectiveness of the derived design method.
$Example1$: When $\bar{\alpha}_i = 1$, consider the continuous networked control system (3) with the matrices taken from[19]:

$$A = \begin{bmatrix} 0 & 1 \\ 1 & -0.1 \end{bmatrix}, BK = \begin{bmatrix} 0 & 0 \\ -0.375 & -1.15 \end{bmatrix},$$
$$C = \begin{bmatrix} 0 & 1 \end{bmatrix}, D = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}$$

For a given scalar $\gamma = 0.87$, $\eta_m = 0$, when using the method of [19], the maximal allowable transmission intervals $\eta_M = 0.8695$. However, when using the Corollary 1 in this paper, the maximal allowable transmission intervals is obtained: $\eta_M = 0.9467$. Obviously, comparing with the approach of [19], using the proposed method in this paper can reduce the conservatism of the system.
$Example2$: Consider an networked control system of (3)

$$A_1 = \begin{bmatrix} 1.1 & -0.24 \\ 0.39 & 0.61 \end{bmatrix}, A_2 = \begin{bmatrix} 0.47 & 0 \\ 0.25 & 0.53 \end{bmatrix},$$
$$B_1 = \begin{bmatrix} 0.6 & 0.1 \\ 0.2 & 0.4 \end{bmatrix}, B_2 = \begin{bmatrix} 0.3 & 0.2 \\ 0.2 & 0.1 \end{bmatrix}, C_1 = \begin{bmatrix} 1 & 2 \\ 0 & 0.5 \end{bmatrix},$$
$$C_2 = \begin{bmatrix} 1 & 0.2 \\ 0 & 1 \end{bmatrix}, D_1 = D_2 = \begin{bmatrix} 0.1 \\ 0.5 \end{bmatrix}$$

When the probabilities of a networked control system staying in two subsystems are known, for a given scalar $\varepsilon_0 = 1$ and $\bar{\alpha}_1 = 0.3$,$\bar{\alpha}_2 = 0.7$,$\eta_m = 0$,$\eta_M = 0.39$, the maximum H_∞ performance is obtained as $\gamma_{min} = 12.07$ by using Theorem 3. Meanwhile, we can obtain the controller feedback gains as follows:

$$K_1 = \begin{bmatrix} -6.9559 & 5.9823 \\ 0.4463 & -5.1865 \end{bmatrix}, K_2 = \begin{bmatrix} 5.2610 & -21.0805 \\ -7.8375 & 16.5099 \end{bmatrix}.$$

For initial state $\phi(t) = [-0.5; 1]$ and disturbance input $\omega(t) = e^{-t}sin(\pi t)$, the state response curves of system (3) and the switching sequence between the two subsystems and can be shown in Figures 1 and 2. From the simulation results, it can be concluded that by using the proposed method, the designed controller feedback gains can guarantee the required system performance of continuous networked control system with time-varying delay.

5 CONCLUSION

In this paper, the H_∞ controller design for continuous networked control system with time-varying delay with a switched system approach is investigated. By constructing a suitable mathematical model of networked control system

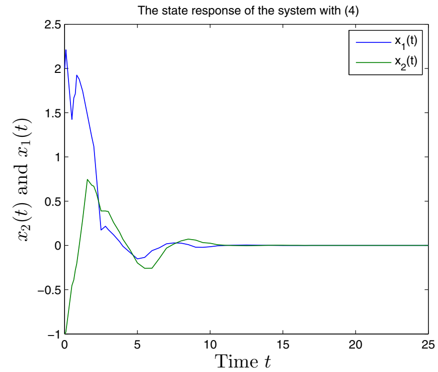

Figure 1: The state response of the system with (3)

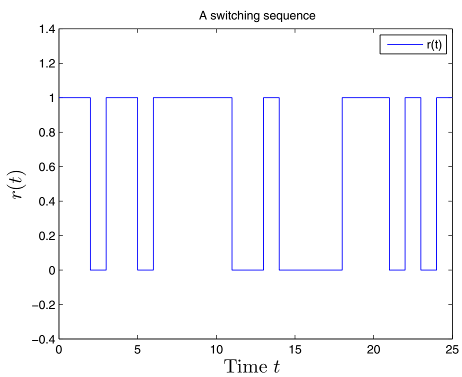

Figure 2: A switching sequence

and using Lyapunov functional method, a H_∞ stabilization criterion is obtained. Meanwhile, the maximum allowable delay of the networked control system with time-varying delay is also obtained. In the end, the effectiveness and correctness of the proposed method is demonstrated by two numerical examples and simulation.

6 ACKNOWLEDGEMENT

This work was supported by the National Natural Science Foundation of China(Grant No. 61473156), Research Fund for the Doctoral Program of Higher Education of China (Grant No. 20133204120018) and the National Science Foundation for Post-doctoral Scientists of China (Grant No. 2014M551487) and Jiangsu Post-doctor Grant(No. 1301009A).

REFERENCES

[1] D. Yue, E. Tian, and Q.-L. Han, "A delay system method for designing event-triggered controllers of networked control systems," *Automatic Control, IEEE Transactions on*, vol. 58, no. 2, pp. 475–481, 2013.

[2] E. Tian, D. Yue, T. C. Yang, Z. Gu, and G. Lu, "T–s fuzzy model-based robust stabilization for networked control systems with probabilistic sensor and actuator failure," *Fuzzy Systems, IEEE Transactions on*, vol. 19, no. 3, pp. 553–561, 2011.

[3] C. Peng and T. C. Yang, "Event-triggered communication and h control co-design for networked control systems," *Automatica*, vol. 49, no. 5, pp. 1326–1332, 2013.

[4] R. Lu, Y. Xu, A. Xue, and J. Zheng, "Networked control with state reset and quantized measurements: observer-based case," *Industrial Electronics, IEEE Transactions on*, vol. 60, no. 11, pp. 5206–5213, 2013.

[5] D. Yue, W. Wong, E. Tian, and T.-C. Yang, "H-infinity filtering for discrete-time switched systems with known sojourn probabilities."

[6] M. Yu, L. Wang, T. Chu, and Q. Fu, "Stabilization of networked control systems with communication constraints," *International Journal of Hybrid Systems*, vol. 4, no. 1, pp. 99–112, 2004.

[7] L. Zhang and E.-K. Boukas, "Stability and stabilization of markovian jump linear systems with partly unknown transition probabilities," *Automatica*, vol. 45, no. 2, pp. 463–468, 2009.

[8] Z. Wen'an, Y. Li, and S. Hongbo, "A switched system approach to networked control systems with time-varying delays," in *Control Conference, 2008. CCC 2008. 27th Chinese*. IEEE, 2008, pp. 424–427.

[9] D. Xie, L. Wang, F. Hao, and G. Xie, "Robust stability analysis and control synthesis for discrete-time uncertain switched systems," in *Decision and Control, 2003. Proceedings. 42nd IEEE Conference on*, vol. 5. IEEE, 2003, pp. 4812–4817.

[10] Y. Song, J. Fan, M. Fei, and T. Yang, "Robust h control of discrete switched system with time delay," *Applied Mathematics and Computation*, vol. 205, no. 1, pp. 159–169, 2008.

[11] W. Qin, Y. Dong, and H. Wang, "Stability analysis of uncertain impulsive switched system with time delay," in *2011 Chinese Control and Decision Conference (CCDC)*, 2011.

[12] D. Xie, L. Wang, F. Hao, and G. Xie, "Lmi approach to l 2-gain analysis and control synthesis of uncertain switched systems," in *Control Theory and Applications, IEE Proceedings-*, vol. 151, no. 1. IET, 2004, pp. 21–28.

[13] Y. Wu, T. Liu, Y. Wu, and Y. Zhang, "H output tracking control for uncertain networked control systems via a switched system approach," *International Journal of Robust and Nonlinear Control*, 2015.

[14] J. Yu, L. Wang, and M. Yu, "Switched system approach to stabilization of networked control systems," *International Journal of Robust and Nonlinear Control*, vol. 21, no. 17, pp. 1925–1946, 2011.

[15] K. Lee and R. Bhattacharya, "Stability analysis of large-scale distributed networked control systems with random communication delays: A switched system approach," *arXiv preprint arXiv:1503.03047*, 2015.

[16] M. Donkers, W. Heemels, N. Van De Wouw, and L. Hetel, "Stability analysis of networked control systems using a switched linear systems approach," *Automatic Control, IEEE Transactions on*, vol. 56, no. 9, pp. 2101–2115, 2011.

[17] A. Seuret and F. Gouaisbaut, "Wirtinger-based integral inequality: application to time-delay systems," *Automatica*, vol. 49, no. 9, pp. 2860–2866, 2013.

[18] C. Peng, D. Yue, E. Tian, and Z. Gu, "A delay distribution based stability analysis and synthesis approach for networked control systems," *Journal of the Franklin Institute*, vol. 346, no. 4, pp. 349–365, 2009.

[19] C. Jeong, P. Park, and S. H. Kim, "Improved approach to robust stability and h performance analysis for systems with an interval time-varying delay," *Applied Mathematics and Computation*, vol. 218, no. 21, pp. 10 533–10 541, 2012.